

SHEAR STRENGTH OF PLATE GIRDERS  
WITH SMALL WEB OPENING

by

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## CHAPTER I. INTRODUCTION

### A. Problem Statement

Openings in the webs of plate girders are necessary to provide for service needs and inspection. An opening will obviously result in a reduction in the web buckling strength and the ultimate strength. The loss of strength can, however, be restored by providing reinforcement.

In 1960 Basler developed a theory for the shear strength of plate girders without web openings. This theory is the basis for the AISC specification provisions for plate girders (1). However, no attempt had been made to extend Basler's theory to the case of plate girders with web openings.

A method of assessing the loss of strength in the web of a plate girder due to an opening will be helpful for the optimum design of such openings. If this loss of strength is unacceptable, only then can the provision of reinforcement around the opening be justified.

### B. Purpose

The purpose of this research is to derive an expression for the shear strength of plate girders with small rectangular openings situated in the center of a panel. The approach is similar to that followed by Basler in developing his shear strength theory for girders without openings, hence, the expression will be referred to as the

Modified Basler equation.

C. Scope

The Modified Basler expression derived herein has the following limitations.

1. The expression for the elastic critical stress is taken from an equation derived by Narayanan (3), termed here the Narayanan equation.

2. The effect of stress concentrations at the corners of the opening is neglected on the assumption that the corners of the opening will be rounded.

3. The comparison of the Modified Basler equation with experimental results is limited to the tests performed by Narayanan (4).

4. The Modified Basler equation is applicable only to small, central, rectangular openings without reinforcement.

## CHAPTER II. REVIEW OF LITERATURE

### A. The Tension Field Concept

The stress redistribution in a plate girder web after web buckling results in the development of tension field action. The action of a girder panel including its framing elements is analogous to that of a Pratt truss panel; that is, a diagonal strip of the web acts as a tension member while the transverse stiffeners act as compression struts. Fig. 2.1(a) illustrates the Pratt truss analogy, whereas yield lines on the test girder (Fig. 2.1(b)) illustrate the development of tension field action in an actual girder (3).

Webs which are stocky enough will not buckle before yielding occurs. In this case all of the shear force will be taken by beam action shear. For girders with slender webs, it is assumed that an applied shear force is carried completely by beam action until the theoretical web buckling stress is reached, and that subsequently, the additional applied shear is carried by tension field action.

### B. Basic Assumptions

Plate girder shear strength theories are derived based upon the assumptions of the classical beam theory. In addition, certain other assumptions were also necessary



(2):

i. the web panel under consideration is supported along its transverse boundaries by adjacent web panels

ii. superposition of stresses resulting from beam and tension field actions is limited by the state of stress that fulfills the yield condition

iii. up to critical load shear is carried in a beam-type manner but the post-buckling contribution comes from the tension field action

iv. the tension field stresses through a web's cross section are uniform.

#### C. Basler's Shear Strength Theory

Basler represented the shear strength of a plate girder as

$$V_u = V_p f(\alpha, \beta, \epsilon_y) \quad (2.1)$$

where,

$\alpha$  = aspect ratio

$\beta$  = slenderness ratio

$V_u$  = ultimate shear force

$V_p$  = shear force for unrestricted shear  
yielding

$f$  = a non-dimensional function

$\epsilon_y$  = strain at yield

The ultimate shear force in a plate girder is a combination of beam and tension field action. Thus

$$V_u = V_T + V_\sigma \quad (2.2)$$

The inclination of the tension field that furnishes the greatest shear component is

$$\phi = \tan^{-1} \sqrt{1 + \alpha^2} - \alpha \quad (2.3)$$

The equation for tension field action is defined as

$$V_{\sigma} = \sigma_t t b \frac{1}{2 \sqrt{1 + \alpha^2}} \quad (2.4)$$

and that for beam shear action is

$$V_{\tau} = V_p \tau_{cr} / \tau_y \quad (2.5)$$

Assuming that Von Mises' yield condition applies, Basler derived the tension field stress  $\sigma_t$  in terms of the yield strength of the material as a ratio  $\sigma_t / \sigma_y$ . After simplifications the ultimate shear formula is expressed in non-dimensional form as

$$V_u / V_p = \tau_{cr} / \tau_y + \frac{\sqrt{3}}{2} \frac{1 - \tau_{cr} / \tau_y}{\sqrt{1 + \alpha^2}} \quad (2.6)$$

#### D. Narayanan's Shear Strength Theory

The ultimate shear capacity of plate girders with webs containing central rectangular holes consists of three components (4):

i. elastic critical load

ii. load carried by tension field action in the post-critical stage

iii. load carried by the flanges when collapse is about to occur due to the formation of plastic hinges.

Narayanan expressed the ultimate shear in a non-dimensional form as

$$V_u/V_p = \frac{\tau_{cr}}{\tau_y} + \left[ \sqrt{3} \sin^2 \theta \left( \cot \theta - \frac{a}{b} \right) - \sqrt{3} \sin \theta \sqrt{\frac{a^2 + b^2}{\tau_y}} \sin(\kappa + \theta) \right] + 4 \sqrt{3} \sin \theta \sqrt{\frac{\sigma_t}{\sigma_y}} M_p^* \quad (2.7)$$

where,

$$M_p^* = \frac{b_f t_f^2 / 4 \sigma_{yf}}{b^2 t \sigma_y} \quad (2.8)$$

$b_f$  = width of flange

$t_f$  = thickness of flange

$\sigma_y$  = yield stress of the web material

$\sigma_{yf}$  = yield strength of the flange material.

The tensile membrane stress is evaluated from Von Mises's yield criterion

$$\frac{\sigma_t}{\sigma_y} = \frac{\sqrt{3}}{2} \frac{\tau_{cr}}{\tau_y} \sin 2\theta + \sqrt{1 + \left( \frac{\tau_{cr}}{\tau_y} \right)^2 \left( \frac{3}{4} \sin^2 2\theta - 1 \right)} \quad (2.9)$$

The approximate equation for computing the value of  $\tau_{cr}$  is

$$\tau_{cr} = K_0 (1 - 1.2 \sqrt{\frac{a_0 b_0}{ab}}) \frac{\pi^2 E}{12 (1 - \nu^2)} \left( \frac{t}{b} \right)^2 \quad (2.10)$$

where,

$$K_{\theta} = 8.98 + 5.6 (b/a)^2 \quad (2.11)$$

The optimum angle of the tension field  $\theta$  is given by

$$\frac{\theta}{\theta_d} = 0.67 - \eta \sqrt{\frac{a \sigma b \sigma}{ab}} \quad (2.12)$$

where,

$\theta_d$  = angle of inclination of panel diagonal

$\eta$  = a non-dimensional coefficient.

## CHAPTER III. DEVELOPMENT OF THE MODIFIED BASLER THEORY

### A. Special Assumptions

The expression for critical shear in this theory, Eq. 2.10, is the approximate equation suggested by Narayanan. However, the value of the shear buckling coefficient used here is the value suggested by AISC (1). The correction was necessary as AISC assumes webs in plate girders to be simply supported unlike Narayanan's fixed support condition.

The critical load term is then

$$\tau_{cr} = K_0 (1 - 1.2 \sqrt{\frac{a_0 b_0}{ab}}) \frac{\pi^2 E}{12 (1 - \nu^2)} \left( \frac{t}{b} \right)^2 \quad (3.1)$$

### B. Tension Field Inclination

Development of the tension field depends on the boundaries of the plate. The flange of a conventionally built plate girder, having very little bending rigidity in the plane of the web cannot effectively resist vertical stresses at its junction with the web. These flanges, thus, do not have any effect on the tension field. However, the tension strips can transmit stresses at the panel boundaries along the transverse stiffeners (2). Hence, only a part of the web contains a significant tension field. The component of this tension field that contributes to shear is (Fig. 3.1(a))

$$\Delta V_{\sigma} = \sigma_t s_o t \sin \phi_o \quad (3.2)$$

where,

$\phi_o$  = tension field inclination

and the strip of tension field that contributes to shear

$$s_o = b \cos \phi_o - a \sin \phi_o - \sqrt{a_o^2 + b_o^2} \sin (\phi_o' + \phi_o) \quad (3.3)$$

From Fig. 3.1(b) it is obvious that the width of this strip depends upon  $\phi_o$ . At the ultimate shear load the value of  $\phi_o$  should produce the greatest shear component of the tension field. This optimum  $\phi_o$  is obtained by differentiating Eq. 3.2 and equating it to zero.

$$\frac{d}{d\phi_o} (\Delta V_{\sigma}) = \frac{d}{d\phi_o} (\sigma_t s_o t \sin \phi_o) = 0 \quad (3.4)$$

$$\text{or} \quad \sigma_t t \left[ \frac{ds_o}{d\phi_o} \sin \phi_o + s_o \cos \phi_o \right] = 0 \quad (3.5)$$

Substitution of the value of  $s_o$  into the above equation gives

$$(b - b_o) \tan^2 \phi_o + 2(a + a_o) \tan \phi_o - (b - b_o) = 0 \quad (3.6)$$

hence,

$$\tan \phi_0 = \sqrt{\left(\frac{a+a_0}{b-b_0}\right)^2 + 1} - \left(\frac{a+a_0}{b-b_0}\right) \quad (3.7)$$

which, in a simplified form, can be written as

$$\tan \phi_0 = \sqrt{(x\alpha)^2 + 1} - x\alpha \quad (3.8)$$

if

$$x = \frac{1 + a_0/a}{1 - b_0/b} \quad (3.9)$$

and

$$\sin \phi_0 = \left[ \frac{1}{2} - \frac{x\alpha}{2\sqrt{(x\alpha)^2 + 1}} \right]^{1/2} \quad (3.10)$$

$$\cos \phi_0 = \left[ 2\sqrt{(x\alpha)^2 + 1} (\sqrt{(x\alpha)^2 + 1} - x\alpha) \right]^{-1/2} \quad (3.11)$$

### C. Tension Field Stress

In a transversely stiffened plate girder the horizontal component of the tension field is transferred to the flange and the stiffeners resist the axial forces (2). For deriving the magnitude of this shear and stiffener force, a succession of equal web panels all subjected to the same shear force is considered (Fig. 3.2(a)).

Fig. 3.2(b) is a free body diagram taken along sections A, B and C. The resultant force acting at the faces A and B can be resolved into a normal component  $F_w$  and a shear component  $V_\sigma$ . However, because of symmetry, the shear

component will be  $V\sigma/2$  per face. The flange force at face B will be higher than that at face A by an amount  $\Delta F_f$ . The stiffener force  $F_s$  and the tension field stresses  $\sigma_t$  act at section C. Considering this figure and the equilibrium equations, the values of these three forces can be obtained.

The equilibrium conditions in the horizontal and vertical directions, respectively, give

$$\Delta F_f = -\sigma_t t \frac{1}{2\sqrt{(x\alpha)^2+1}} \left[ a - a_0 - \frac{b_0}{\sqrt{(x\alpha)^2+1} - x\alpha} \right] \quad (3.12)$$

$$F_s = \sigma_t t \frac{1}{2\sqrt{(x\alpha)^2+1}} \left[ (a - a_0)(\sqrt{(x\alpha)^2+1} - x\alpha) - b_0 \right] \quad (3.13)$$

and moment about O gives

$$V\sigma = \sigma_t t \frac{b}{2\sqrt{(x\alpha)^2+1}} \left[ 1 - \frac{a_0}{a} - \frac{b_0/a}{\sqrt{(x\alpha)^2+1} - x\alpha} \right] \quad (3.14)$$

Eq. 3.14, which is the tension field equation, can be expressed in terms of non-dimensional ratios as

$$V\sigma = \sigma_t t \frac{b}{2\sqrt{(x\alpha)^2+1}} \left[ 1 - \frac{\alpha_0 Y_0}{\alpha} - \frac{1}{\sqrt{(x\alpha)^2+1} - x\alpha} \right] \quad (3.15)$$

#### D. Critical Shear Stress

The critical load term is represented in Eq. 3.1. However, in order to account for strain-hardening within the slip bands in mild steel, Basler recommended that the value of  $\tau_{cr}$  be taken to be



$$\tau_{cr} \quad \text{when} \quad \tau_{cr} \leq 0.8 \tau_y$$

$$\text{and} \quad \tau_{cr} = \sqrt{0.8 \tau_y \tau_{cr}} \quad \text{when} \quad \tau_{cr} > 0.8 \tau_y$$

#### E. Ultimate Shear Force

The full plastic shear force is attained when yielding occurs throughout the web depth. Hence,

$$V_p = \tau_y b t \quad (3.16)$$

From Von Mises's theory  $\tau_y = \frac{\sigma_y}{\sqrt{3}}$ . Thus,

$$V_p = \frac{1}{\sqrt{3}} \sigma_y b t \quad (3.17)$$

The critical shear

$$V_{cr} = V_p \frac{\tau_{cr}}{\tau_y} \quad (3.18)$$

In Eq. 3.18 the effect of the opening is assumed to be taken care of by the already reduced critical shear stress term  $\tau_{cr}$ .

It has been already mentioned that the ultimate shear force  $V_u$  is a combination of beam and tension field action. Hence,

$$V_u = V_T + V_\sigma \quad (3.19)$$

and

$$V_u = V_p \frac{\tau_{cr}}{\tau_y} + \sigma_t \frac{V_p}{\tau_y} \frac{1}{2\sqrt{(\chi\alpha)^2 + 1}} \left( 1 - \frac{\alpha_o \gamma_o}{\alpha} - \frac{\gamma_o/\alpha}{\sqrt{(\chi\alpha)^2 + 1} - \chi\alpha} \right) \quad (3.20)$$

or

$$\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \frac{\sigma_t}{\sigma_y} \frac{1}{2\sqrt{(\chi\alpha)^2 + 1}} \left( 1 - \frac{\alpha_o \gamma_o}{\alpha} - \frac{\gamma_o/\alpha}{\sqrt{(\chi\alpha)^2 + 1} - \chi\alpha} \right) \quad (3.21)$$

Considering Basler's simplification

$$\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \frac{1 - \tau_{cr}/\tau_y}{\sqrt{(\chi\alpha)^2 + 1}} \left( 1 - \frac{\alpha_o \gamma_o}{\alpha} - \frac{\gamma_o/\alpha}{\sqrt{(\chi\alpha)^2 + 1} - \chi\alpha} \right) \quad (3.22)$$

Eq. 3.22 is the Modified Basler equation for computing the shear strength of plate girders containing small, central, rectangular web openings. The term mainly responsible for decreasing the shear strength is

$$\left( 1 - \frac{\alpha_o \gamma_o}{\alpha} - \frac{\gamma_o/\alpha}{\sqrt{(\chi\alpha)^2 + 1} - \chi\alpha} \right) \quad (3.23)$$

## CHAPTER IV. EFFECT OF OPENING SIZE ON SHEAR STRENGTH

### A. Tension Field Limits

The contribution of the tension field force to the panel shear strength is controlled by the parameters  $\alpha, \alpha_0$  and  $\gamma_0$ . For certain values of these parameters there must be a condition when there is no contribution of this force to the panel shear strength. This is possible when Eq. 3.23, that is,

$$1 - \frac{\alpha_0 \gamma_0}{\alpha} - \frac{\gamma_0 / \alpha}{\sqrt{(x\alpha)^2 + 1}} - x\alpha = 0$$

or

$$\frac{\alpha_0 \gamma_0}{\alpha} + \frac{\gamma_0 / \alpha}{\sqrt{(x\alpha)^2 + 1}} - x\alpha = 1 \quad (4.1)$$

and in terms of dimensionless ratios

$$\frac{\alpha_0 \gamma_0}{\alpha} - \frac{1}{\sqrt{\left[\left(\frac{1 + \alpha_0 \gamma_0 / \alpha}{1 - \gamma_0}\right) \alpha\right]^2 + 1}} - \left(\frac{1 + \alpha_0 \gamma_0 / \alpha}{1 - \gamma_0}\right) \alpha = 1 \quad (4.2)$$

Of the parameters involved in Eq. 4.2,  $\alpha$  is usually defined for a plate girder problem. Considering a value of either  $\alpha_0$  or  $\gamma_0$ , a value of the other parameter can be obtained from this equation through a process of iteration.

Fig. 4.1 is a plot for various values of these parameters satisfying Eq. 4.2. From this plot, the value of a parameter can be found if that of the other two are known.

The curves in this figure represents the zero tension field points, and the areas above and below them mark the regions for which the tension field force contribution to the panel shear strength is negative or positive, respectively. Hence, in order to obtain a positive effect of the tension field on the shear strength of a plate girder, the strength parameters must have values that fall in the region lying below these curves.

### B. Opening Size

From Eq. 4.2 it is obvious that the maximum size of the opening cannot be defined with the help of just one of the parameters independently. They are all interrelated and a change in the value of one necessitates a change in the values of the other two in order to satisfy the equation.

Size of the opening will be defined, in this paper, for different aspect ratios in terms of  $\alpha_o$  and  $\gamma_o$ . Eq. 4.2 can be rearranged to

$$2\left(\frac{\alpha + \gamma_o \alpha_o}{1 - \gamma_o}\right)\left(\frac{\gamma_o}{\alpha - \alpha_o \gamma_o}\right) + \left(\frac{\gamma_o}{\alpha - \alpha_o \gamma_o}\right)^2 = 1 \quad (4.3)$$

From Eq. 4.3, assuming a value of  $\alpha$  and  $\alpha_o$ , a value of  $\gamma_o$  can be obtained. Proceeding in this manner, the values of  $\alpha_o$  and  $\gamma_o$  which satisfy the condition that the contribution of tension field force to the panel shear strength is positive, can be obtained for different aspect ratios. The values of these ratios thus obtained is

presented in Table 4.1.

The area of the opening can also be expressed as a percentage of the area of the web panel from the expression

$$a_o b_o = \frac{\alpha_o \gamma_o}{\alpha} a b \quad (4.4)$$

## CHAPTER V. RELATIONSHIP BETWEEN BASLER AND THE MODIFIED BASLER FORMULAS

### A. Comparison Of The Two Formulas

Inclination of tension stresses according to the Basler and the Modified Basler theories are represented by Eqs. 2.3 and Eq. 3.8, respectively. These equations are similar except for the term X, whose value is given in Eq. 3.9. Accordingly, the sine and cosine values of the angle required in determining the tension field force also change.

Critical web buckling in the Modified Basler theory differs from that of the Basler theory by the amount

$$1 - 1.2 \sqrt{\frac{a_o b_o}{a b}} \quad (5.1)$$

The tension field contribution to the panel shear strength in the Modified Basler theory incorporates additional terms that are shown in Eqs. 3.9 and 3.23.

### B. Analytical Observations

1. The inclination of tension stresses is greater in the Modified Basler formula for all combinations of aspect ratios, opening widths and depths (Table 5.1 and Fig. 5.1 to 5.2).

2. The panel shear strength according to Basler's formula is always on the higher side (Table 5.2 and

Fig. 5.3 to 5.8).

3. The difference in shear strength as given by the two theories increases with an increase in the size of the opening. However the difference is more pronounced when the depth of the opening rather than its width increases.

4. For similar opening sizes, this difference decreases as the aspect ratio increases.

5. The contribution of the tension field force to shear strength is greater for larger values of slenderness, for any particular value of aspect ratio (Table 5.3).

### C. Discussion

The term  $X$  in Eq. 3.8 will always be greater than one. Therefore, this inclination of tension stresses is always a lower value than that given by Eq. 2.3. This term also reduces the tension field contribution in Eq. 3.22.

Incorporation of Eq 5.1 in the critical web buckling equation lowers its contribution to the panel shear strength. The term mainly responsible in reducing the tension field forces is represented by Eq. 3.23. An increase in the value of  $\alpha_0$  or  $\gamma_0$  decreases this term. However,  $\gamma_0$  affects the equation twice. Moreover,  $X$  increases proportionately more for an increase in  $\gamma_0$  rather than for  $\alpha_0$ .

As  $\kappa$  increases the negative terms in Eq. 3.23 automatically decreases. An increased  $\kappa$  also decreases the

value of Eq. 3.22. However, the effect of its increase is more pronounced in Eq. 3.23.

An increase in slenderness decreases the critical shear, thereby increasing the shear contribution of the tensile forces.



## CHAPTER VI. COMPARISON OF THE MODIFIED BASLER AND NARAYANAN'S FORMULAS

### A. Comparative Study Of The Two Formulas

The component due to web buckling in both the Modified Basler and Narayanan's theory is similar except for the shear buckling coefficient. Parameters involved in the equations for the tension field stress inclination are the same although they are related to each other in different orders.

Narayanan's tension field contribution equation is expressed in terms of the ratio  $\sigma_t/\sigma_y$ , whereas Basler's simplified approach is applied to the Modified Basler theory. The second term in Eq. 2.7 is the tension field contribution, where the reduction factor is accounted for by the negative term incorporating  $a_o$  and  $b_o$ . The third term in Eq. 2.7 represents the contribution by the flanges. This flange contribution is neglected in the Modified Basler theory.

### B. Analytical Observations

1. The tension stress inclination according to Narayanan's theory gives a higher value than that given by the Modified Basler theory (Table 5.1 and Fig. 5.1 to 5.2).

2. Narayanan's formula always gives a relatively higher value of panel shear strength (Table 5.2 and Fig. 5.3 to 5.8).

3. For values of  $\gamma_0$  up to about 0.1, the shear strengths given by the two theories do not differ much.

4. As  $\alpha$  increases, this difference decreases for any value of  $\alpha_0$  or  $\gamma_0$ .

5. The difference is more pronounced when  $\gamma_0$  increases rather than  $\alpha_0$ .

6. The shear contribution from the tension field forces as calculated from these two equations does not differ very much (Table 5.3. and Fig. 5.9 to 5.10).

### C. Discussion

Narayanan's theory involves a higher value of  $k_0$ . It also considers an additional contribution from the flanges. For any value of the opening parameters the web buckling contribution is significantly higher according to this theory. This coupled with the flange contribution always gives a higher value of shear strength.

The theoretical predicted values from Narayanan's equation are in close agreement with those obtained from the test results. This is not the case with the Modified Basler equation. However, when the lower value of the coefficient of buckling is taken and the flange effect is neglected in Narayanan's equation, the predicted values differ considerably from the test values (Table 5.5 and Fig. 5.11).

An increase in the value of  $\gamma_0$  decreases the tension field inclination. This decreases the negative term in Eq. 2.7 relatively more than the other term in the square

bracket. So, the tension field contribution is not reduced to a large extent. However, Eq. 3.21 depends more heavily on  $\gamma_0$ . The tension field inclination also depends on  $\alpha_0$ , but the dependence of Eq. 3.21 on it is not as heavy as in the previous case. This explains why  $\gamma_0$  affects the Modified Basler theory more than Narayanan's theory.

An increase in  $\alpha$  does not affect Narayanan's theory very much. However, this increases the panel shear strength in the case of Eq. 3.22 by reducing the difference predicted by the two theories.

For the purpose of comparing the behaviour of the two theories under identical conditions, Narayanan's equation is modified to the case where the flange effect is neglected and the value of  $k_0$  used is the AISC recommended value. The values of ultimate shear force calculated from these equations do not differ much when the aspect ratio is one or larger. When the aspect ratio is less than one the difference increases only when opening depth is about 20% of the girder depth (Table 5.4, And 5.12).

Since the tension field forces predicted by these two theories do not vary by any significant amount it can be stated that the error for not considering the effect of stress concentration while deriving this theory is negligible.

## CHAPTER VII. SUMMARY AND CONCLUSIONS

### A. Summary

A method of assessing the ultimate shear strength of plate girders with small, central rectangular openings has been developed. Analytical calculations show that for larger aspect ratios and smaller depths of opening this theory compares satisfactorily with Narayanan's theory. The differences in the values of shear forces according to these two theories are mainly due to the different nature of boundary conditions assumed.

### B. Conclusions

1. Compared with test results the Modified Basler equation gives conservative results. This is mainly due to the fact that the effect of the flanges is neglected and a lower value of the web buckling coefficient is used. Hence, for this problem the AISC specifications regarding these two factors is not very practical.

2. In Modified Basler theory the loss of shear strength is relatively greater when  $\gamma_0$  increases rather than when  $\alpha_0$  increases.

3. As the aspect ratio increases the relative differences in shear predicted by the Modified Basler and Narayanan's equations decreases.

4. The tension field force is non-zero only when

the opening depth is about 20% of the web panel depth or less.

5. Depending upon aspect ratio and the opening parameters the Modified Basler theory is applicable to openings with areas as large as 16% of the area of the web panel.

6. Results predicted by the Modified Basler theory is conservative compared to those predicted by Narayanan's theory.

### C. Recommendations

i. The tests conducted by Narayanan are not sufficient for evaluating the Modified Basler theory. Hence, more tests should be conducted for values of  $\alpha$ ,  $\alpha_0$  and  $\gamma_0$  which defines a positive tension field contribution.

ii. An attempt could be made to include the contribution of shear from the flanges in the Modified Basler theory.

iii. Effort should be made to extend the Modified Basler theory to web panels with off-center openings.

iv. A method of designing reinforcement for such girders should be developed.

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## APPENDIX A

### Notation

- $a$  = spacing of the tranverse stiffeners
- $a_o$  = width of opening
- $b$  = depth of girder web
- $b_f$  = width of flange plate
- $b_o$  = depth of opening
- $E$  = modulus of elasticity
- $k_o$  = buckling coefficient
- $M_f^*$  = flange stiffness parameter
- $s_e$  = effective width of tension field
- $t$  = thickness of web
- $t_f$  = thickness of flange plate
- $\alpha$  = aspect ratio,  $(a/b)$
- $\alpha_o = a_o/b_o$
- $\gamma_o = b_o/b$
- $\beta$  = slenderness ratio,  $(b/t)$
- $\nu$  = Poisson's ratio,  $(0.3)$
- $\sigma$  = normal stress
- $\tau$  = shear stress
- $\phi$  = Basler's tension field inclination
- $\phi_o$  = modified Basler tension field inclination
- $\sigma_t$  = tensile membrane stress
- $\theta$  = angle of inclination of tensile membrane stress
- $\phi_o'$  = angle of inclination of opening

## Subscripts

B = Basler

cr = critical

f = flange

MB = modified Basler

N = Narayanan

o = opening

t = tension

u = ultimate

w = web

y = yielding

$\sigma$  = as carried in tension

$\tau$  = as carried in shear



## APPENDIX B

### References

1. Manual of Steel Construction, American Institute of Steel Construction.
2. "Strength of Plate Girders in Shear," by Konrad Basler, Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. ST 7, October, 1961, pp 151-180.
3. "Plate Girders," by Cooper P. B., Structural Steel Design, The Ronald Press Company, New York, 1964.
4. "Design of Slender Webs Having Rectangular Holes," by Rangachari Narayanan and Norire Gara-Verni Der-Avanessian, Journal of Structural Engineering, American Society of Civil Engineers, Vol. 111 No. 4. Apr. 1985, pp. 777-787.
5. "Equilibrium Solutions for Predicting the Strength of Webs with Rectangular Holes," by Narayanan, R., and Der-Avanessian, N. G. V., Proceedings of the Institution of Civil Engineers, London, England, Part 2, Vol. 75. June, 1983, pp. 265-282.

```

10 REM PROGRAM TO FIND THE ANGLE OF INCLINATION OF TENSION
    FIELD
20 INPUT "NO. OF ASPECT RATIOS (ALPHA) N";N
30 FOR I=1 TO N
40 READ ALPHA(I)
50 NEXT I
60 DATA 0.5,1.0,1.5,2.0,2.5,3.0
70 INPUT "no. of gma j";J
80 FOR K=1 TO J
90 READ GMA(K)
100 NEXT K
110 DATA .1,.2,.3,.4,.5
120 INPUT "no. of alfo m";M
130 FOR L=1 TO M
140 READ ALFO(L)
150 NEXT L
160 DATA .5,1.,1.5,2.
170 LPRINT "alpha      bo/b      ao/bo      phib      phimb
    phinar"
180 FOR I=1 TO N
190 FOR K=1 TO J
200 FOR L=1 TO M
210 FIR=SQR(ALPHA(I)^2+1)-ALPHA(I)
220 FID=FIR*180/3.14
230 A=(ALPHA(I)+GMA(K)*ALFO(L))/(1-GMA(K))
240 B=SQR(A^2+1)
250 C=B-A
260 PHIR=ATN(C)
270 PHID=PHIR*180/3.14
280 D=ATN(1/ALPHA(I))
290 E=ALFO(L)*GMA(K)^2/ALPHA(I)
300 F=.65*SQR(E)
310 THI=.67*D-F*D
320 THIR=ATN(THI)
330 THID=THIR*180/3.14
340 LPRINT ALPHA(I);"      ";GMA(K);"      ";ALFO(L);"
    ";FID;"      ";PHID;"      ";THID
350 NEXT L
360 NEXT K
370 NEXT I

```

```

10 REM PROGRAM TO FIND THE ULTIMATE SHEAR RATIOS
20 REM *****
30 REM User's Manual
40 REM In the input statement defining type, use type=2.
50 REM This will give the shear according to Basler's
60 REM theory. The program then runs to give shear
70 REM according to the Modified Basler and Narayanan's
75 REM theories respectively.
80 REM *****
90 LPRINT "NO. OF ASPECT RATIOS"
100 INPUT "NO. OF ASPECT RATIOS (ALPHA) N";N
110 LPRINT N
120 LPRINT "ASPECT RATIO"
130 DIM ALPHA(N), GMA(10), ALFO(10), CS(N,18),
    CSO(N,200),CSO1(N,5,200), CSN1(N,5,200), C(200),
    CSN(N,200)
140 FOR I=1 TO N
150 READ ALPHA(I)
160 LPRINT ALPHA(I)
170 NEXT I
180 DATA .5,1,1.5,2
190 PI = 3.14
200 E = 29000
210 NU = .3
220 SIG = 36
230 BF = 14
240 TF = 1
250 TW = .2
260 INPUT "OPENING ? IF YES TYPE 1, IF NO TYPE 2";TYPE
270 IF TYPE = 1 THEN LPRINT"SHEAR BY BASLER'S EQUATION
    MODIFIED FOR OPENING": GOTO 320
280 IF TYPE = 3 THEN LPRINT"SHEAR BY NARAYANANAN EQUATION
    (OPENING)": GOTO 310
290 O = 1: W=1:LPRINT"SHEAR BY BASLER EQUATION
    (NO OPENING)"
300 LPRINT "ALPHA BTA Vu/Vp": GOTO 550
310 IF TYPE=3 THEN CLS:PRINT"CALCULATIONS BY NARAYANAN'S
    EQU."
320 INPUT "NO.OF OPENING DEPTHS O";O
330 LPRINT "bo/b (gamma)"
340 FOR J=1 TO O
350 READ GMA(J)
360 LPRINT GMA(J)
370 NEXT J
380 DATA .1,.2,.3,.4,.5
390 DATA 2
400 INPUT "NO. OF OPENING WIDTHS W";W
410 LPRINT "ao/bo (alpha opening)"
420 FOR Q=1 TO W
430 READ ALFO(Q)
440 LPRINT ALFO(Q)
450 NEXT Q

```

```

460 DATA .1..2..3..4..5
470 DATA 2
480 IF TYPE = 3 THEN LPRINT"LAST COLUMN GIVES SHEAR RATIO
CALCULATED BY MODIFIED BASLER EQU. OVER NARAYANAN'S EQU"
490 IF TYPE = 3 THEN LPRINT"ALPHA      bo/b      ao/bo
      BTA      VU/VP      CSO/CSN"
500 REM TC=TAU CRITICAL
510 REM TY=TAU YIELD
520 IF TYPE = 3 THEN GOTO 550
530 LPRINT "THE LAST COLUMN GIVES THE SHEAR RATIO
CALCULATED BY MOD. BASLER EQU. OVER BASLER EQU."
540 LPRINT"ALPHA      bo/b      ao/bo      BTA      VU/VP
      CSO/CS
550 FOR I=1 TO N
560 PT = 0
570 FOR J=1 TO O
580 FOR Q=1 TO W
590 PT = PT+1
600 PR = 0
610 FOR Z = 1 TO 18
620 PR = PR+1
630 BTA = 20 + (Z-1)*20
640 C(Z)=PI^2*E/(12*(1-NU^2))*(1/BTA)^2
650 IF TYPE = 3 THEN GOTO 890
660 X=(1+ALFO(Q)*GMA(J)/ALPHA(I))/(1-GMA(J))
670 P=(SQR((ALPHA(I)*X)^2+1))-ALPHA(I)*X
680 T=TAN(P)
690 TANPHI=T
700 IF ALPHA(I)<((ALFO(Q)*GMA(J))+(GMA(J)/TANPHI))
      GOTO 1080
710 TY=SIG/SQR(3)
720 TCRI0=(5+5/ALPHA(I)^2)*C(Z)
730 KO=(5+5/ALPHA(I)^2)*(1-1.2*SQR(GMA(J)^2*ALFO(Q)/
      ALPHA(I)))
740 IF TCRI0<=.8*TY THEN TCO=KO*C(Z)
750 IF TCRI0 > .8*TY THEN TCO=SQR(.8*TY*KO*C(Z))
760 V=(1-ALFO(Q)*GMA(J)/ALPHA(I)-(GMA(J)/ALPHA(I))/P)
770 Y=SQR(3)/2*(1-TCO/TY)/SQR((ALPHA(I)*X)^2+1)
780 REM CSO=CRITICAL SHEAR (WUTH OPENING)
790 IF TYPE =1 THEN GOTO 830
800 IF TCO<TY THEN CS(I,PR)=TCO/TY+Y*V
810 IF TCO>TY THEN CS(I,PR)=TCO/TY
820 LPRINT;ALPHA(I);"      ";BTA;"      ";CS(I,PR):
      GOTO 1080
830 IF TCO<TY THEN CSO(I,PT)=TCO/TY+Y*V
840 IF TCO>TY THEN CSO(I,PT)=TCO/TY
850 RATIO = CSO(I,PT)/CS(I,PR)
860 CSO1(I,PT,Z)= CSO(I,PT)
870 REM CSO = CRITICAL SHEAR (WITH OPENING)
880 LPRINT;ALPHA(I);"      ";GMA(J);"      ";ALFO(Q);"
      ";BTA;"      ";CSO(I,PT);"      ";RATIO: GOTO 1080
890 IF BTA < 200 GOTO 1080
900 THN = .67*ATN(1/ALPHA(I)) -

```

```

      .65*SQR(GMA(J)^2*ALFO(Q)/ALPHA(I))* .67/ALPHA(I)
910 REM THN = THITA NARAYANANAN
920 ADN = ATN (1/ALFO(Q))
930 REM ADN = ANGLE OF INCLINATION OF THE OPENING'S
      DIAGONAL
940 TCN = (8.979999+5.6/ALPHA(I)^2) * (1-1.2*SQR(GMA(J)^2*
      ALFO(Q)/ALPHA(I)))*C(Z)
950 REM TCN = NARAYANANAN CRITICAL SHEAR
960 SN = SIN(THN)
970 SN2 = SIN(2*THN)
980 RSTRESS =-SQR(3)/2*CN/TY*SN2+SQR(1+(TCN/TY)^2*
      (3/4*SN2^2-1))
990 REM RSTRESS = RATIO OF TENSILE MEMBRANE STRESS TO
      YIELD STRESS OF WEB
1000 TN = TAN(THN)
1010 CT = 1/TN
1020 SAN = SIN(ADN+THN)
1030 MP = (BF*TF^2/(4*SIG))/((BTA*TW)^2*TW*SIG)
1040 CSN(I,PT) =TCN/TY+(SQR(3)*SN^2*(CT-ALPHA(I))*RSTRESS-
      SQR(3)*SN*SQR(((ALFO(Q)*GMA(J)*BTA*TW)^2+
      (GMA(J)*BTA*TW)^2)/(BTA*TW)^2)*RSTRESS*SAN)+
      4*SQR(3)*SN*SQR(RSTRESS*MP)
1050 CSN1(I,PT,Z)=CSN(I,PT)
1060 BOVERN = CSO1(I,PT,Z)/CSN1(I,PT,Z)
1070 LPRINT:ALPHA(I);"      ";GMA(J);"      ";ALFO(Q);"
      ";BTA;"      ";CSN(I,PT);"      ";BOVERN
1080 NEXT Z
1090 NEXT Q
1100 NEXT J
1110 PT1(I) = PT
1120 NEXT I
1130 IF TYPE = 2 THEN TYPE = 1: GOTO 270
1140 IF TYPE = 1 THEN TYPE = 3: GOTO 280

```

```

C PROGRAM TO PLOT THE ULTIMATE SHEARS USING THE FORTRAN
PLOTTER
//**++VMMMSG LOG TIME 3.00 PRINT REGION 1000K
/*PLOTTER
//PSHEAR EXEC PLOTVCLG
//FORT.SYSIN DD *
      REAL KOI,KO,NU
      INTEGER O,W
      DIMENSION ALPHA(10), GMA(10), ALFO(10), CS(10,18),
1 CSN(10,100), CSO(10,100), C(100), PT1(10), SCN(9),
1 S1(15), S2(15), S3(15), B(10)
      READ(5,*)N
      DO 10 I=1,N
        READ(5,*) ALPHA(N)
10      CONTINUE
        PI=3.14
        E=29000.
        NU=.3
        SIG=36.
        TF=.315
        BF=3.94
        TW=.079
        READ(5,*)NO
        IF(NO.EQ.2) GOTO 60
50      READ(5,*)O,W
        DO 20 J=1,O
          READ(5,*) GMA(J)
20      CONTINUE
        DO 30 Q=1,W
          READ(5,*)ALFO(Q)
30      CONTINUE
        GOTO 100
60      O=1
        W=1
        GOTO 100
100     DO 110 I=1,N
          PT=0.
          DO 120 J=1,O
            DO 130 Q=1,W
              PT=PT+1.
              PR=0.
              DO 140 Z=1,9
                PR=PR+1.
                BTA=200.+(Z-1)*20.
                C(Z)=PI**2*E/(12.*(1.-NU**2))*(1./BTA)**2
                B(Z)=BTA
                TY=SIG/SQRT(3.)
                IF(NO.EQ.3) GOTO 40
                X=(1.+ALFO(Q)*GMA(J)/ALPHA(I))/(1.-GMA(J))
                P=SQRT((ALPHA(I)*X)**2+1.)-ALPHA(I)*X
                PHIO=ATAN(P)
                S2PHIO=SIN(2.*PHIO)
              40
            130
          120
        110

```

TANPHI=T

```

1 IF(1.LE.(ALFO(Q)*GMA(J)/ALPHA(I)+GMA(J)/ALPHA(I)
  )/P)) GOTO 130
  KOI=(5+5/ALPHA(I)**2))
  TCRI0=KOI*C(Z)
1 KO=KOI*(1.-1.2*(SQRT(GMA(J)**2*ALFO(Q)/
  ALPHA(I))))
  IF(TCRI0.LE..8*TY) THEN
    TCO=KO*C(Z)
  ELSE
    TCO=SQRT(.8*TY*KO*C(Z))
  END IF
1 V=(1.-ALFO(Q)*GMA(J)/ALPHA(I)-GMA(J)/ALPHA(I)
  )/P)
1 Y=SQRT(3.)/2.*(1.-TCO/TY)/SQRT((ALPHA(I)*X)**2
  +1.)
  IF(NO.EQ.2) THEN
    IF(TCO.LE.TY) THEN
      CS(I,PR)=TCO/TY+Y*V
    ELSE
      CS(I,PR)=TCO/TY
    END IF
    S1(Z)=CS(I,PR)
  ELSE IF(TCO.LT.TY) THEN
    CSO(I,PT)=TCO/TY+V*Y
  ELSE
    CSO(I,PT)=TCO/TY
  END IF
  S2(Z)=CSO(I,PT)
  IF(NO.EQ.2) GOTO 140
40 THN=.67*ATAN(1/ALPHA(I))- .65*(SQRT(GMA(J)**2*
  1 ALFO(Q)/ALPHA(I)))*ATAN(1/ALPHA(I))
  ADN=ATAN(1/ALFO(Q))
  TCN=(8.98+5.6/ALPHA(I)**2)*(1.-1.2*SQRT(GMA(J)**2
  1 *ALFO(Q)/ALPHA(I)))*C(Z)
  SCN(Z)=TCN/TY
  SN=SIN(THN)
  SN2=SIN(2*THN)
  RSTRES=-(SQRT(3.)/2.*TCN/TY*SN2)+SQRT(1.+(TCN/TY)
  1 **2*(3./4.*SN2**2-1.))
  TN=TAN(THN)
  CT=1./TN
  PM=BF*TF**2/4.*SIG/((BTA*TW)**2*TW*SIG)
  CSN(I,PT)=TCN/TY+(SQRT(3.)*SN**2*(CT-ALPHA(I)*
  1 RSTRES-SQRT(3.)*SN*SQRT(((ALFO(Q)*GMA(J)*BTA*TW)**2
  1 +(GMA(J)*BTA*TW**2)/(BTA*TW)**2)*RSTRES*SN)+
  1 4.*SQRT(3.)*SN*SQRT(RSTRES*PM)
  S3(Z)=CSN(I,PT)
140 CONTINUE
130 CONTINUE
120 CONTINUE
  PT1(I)=PT

```

```

IF(NO.EQ.2)THEN
  NO=1
  GOTO 50
END IF
IF(NO.EQ.1) THEN
  NO=3
  GOTO 75
END IF
S1(10)=0.
S1(11)=.2
S2(10)=0.
S2(11)=.2
S3(10)=0.
S3(11)=.2
B(10)=160.
B(11)=20.
CALL PLOTS
CALL PLOT (0.,3.,23)
CALL FACTOR(.65)
CALL AXIS (0.,0., 'BETA', -4,10.,0,B(10),B(11))
CALL AXIS (0.,0., 'VU/VP', 5,6.,90.,S1(10),S1(11))
CALL LINE (B,S1,9,1,0,0)
CALL LINE (B,S2,9,1,+1,26)
CALL DASHLN (B,S3,9,1)
CALL SYMBOL (3.3,5.5,.14, 'ALPHA=  , GAMMA(0)=  ,
1 ALPHA(0)= ',0.32)
CALL PLOT (0.,0.,999)
STOP
END
//GO.SYSIN DD *

DATA

/*

```



Table 4.1 - No Tension Field Condition

$\alpha$	$\alpha_0$			
	0.5	1.0	1.5	2.0
0.5	0.2	0.163	0.138	0.121
1.0	0.260	0.225	0.199	0.180
1.5	0.283	0.255	0.231	0.212
2.0	0.296	0.271	0.250	0.233
2.5	0.303	0.281	0.263	0.249
3.0	0.308	0.289	0.273	0.259

Note: Table contains  $\gamma_0$  values for respective  $\alpha$  and  $\alpha_0$

Table 5.1 - Angle of Tension Field Inclination

$\alpha_0$	$\alpha$	$\gamma_0 = 0.1$			$\gamma_0 = 0.2$		
0.5	0.5	35.43	29.30	33.83	35.43	26.58	30.89
	1.0	23.74	20.31	26.12	23.74	18.02	24.43
	1.5	17.36	15.08	20.41	17.36	13.29	19.29
	2.0	13.53	11.86	16.47	13.53	11.13	15.68
1.0	0.5	35.43	28.17	32.64	35.43	24.42	28.31
	1.0	23.74	19.65	25.43	23.74	16.85	22.99
	1.5	17.36	14.69	19.95	17.36	12.61	18.35
	2.0	13.53	11.61	16.15	13.53	10.00	15.10
1.5	0.5	35.43	27.09	31.70	35.43	22.51	26.23
	1.0	23.74	19.03	24.89	23.74	15.81	21.87
	1.5	17.35	14.31	19.59	17.35	11.99	17.62
	2.0	13.53	11.36	15.89	13.53	9.59	14.50
2.0	0.5	35.43	26.07	30.89	35.43	20.83	24.43
	1.0	23.74	18.44	24.43	23.74	14.88	20.91
	1.5	17.35	13.96	19.29	17.35	11.43	17.00
	2.0	13.53	11.13	15.68	13.53	9.22	14.06

Table 5.2 - Ultimate Shear Force

$\beta$	$\alpha$	$\gamma_0$	$\alpha_0$	$(V_u/V_p)_B$	$(V_u/V_p)_{MB}$	$(V_u/V_p)_N$
200	0.5	0.1	0.5	0.952	0.816	1.046
360				0.829	0.530	0.714
200	0.5	0.2	0.5	0.952	0.598	0.991
360				0.829	0.185	0.622
200	0.5	0.1	2.0	0.952	0.651	0.998
360				0.829	0.291	0.643
200	0.5	0.2	2.0	0.952	0.120	0.746
360				0.892		0.488
200	2.0	0.1	0.5	0.508	0.394	0.544
360				0.425	0.299	0.298
200	2.0	0.2	0.5	0.508	0.277	0.486
360				0.425	0.172	0.248
200	2.0	0.1	2.0	0.508	0.348	0.513
360				0.424	0.254	0.279
200	2.0	0.2	2.0	0.508	0.192	0.429
360				0.424	0.094	0.217

Table 5.3 - Tension Field Force

$\beta$	$\alpha$	$\gamma_0$	$\alpha_0$	$(V\sigma)_B$	$(V\sigma)_{MB}$	$(V\sigma)_N$
200	0.5	0.1	0.5	0.165	0.123	0.059
360				0.586	0.316	0.299
200	0.5	0.2	0.5	0.165		0.092
360				0.586		0.253
200	0.5	0.1	2.0	0.165	0.052	0.099
360				0.586	0.106	0.274
200	0.5	0.2	2.0	0.165		0.142
360				0.586		0.218
200	2.0	0.1	0.5	0.311	0.209	0.108
360				0.364	0.242	0.127
200	2.0	0.2	0.5	0.311	0.103	0.075
360				0.364	0.119	0.086
200	2.0	0.1	2.0	0.311	0.175	0.102
360				0.364	0.200	0.117
200	2.0	0.2	2.0	0.311	0.043	0.067
360				0.364	0.048	0.066

Table 5.4 - Ultimate Shear Force

$\beta$	$\alpha$	$\gamma_0$	$\alpha_0$	$(V_u/V_p)_B$	$(V_u/V_p)_{MB}$	$(V_u/V_p)_N$
200	0.5	0.1	0.5	0.952	0.816	0.827
360				0.829	0.530	0.532
200	0.5	0.2	0.5	0.952	0.598	0.741
360				0.829	0.185	0.670
200	0.5	0.1	2.0	0.952	0.651	0.753
360				0.829	0.291	0.473
200	0.5	0.2	2.0	0.952	0.120	0.576
360				0.892		0.350
200	2.0	0.1	0.5	0.508	0.394	0.304
360				0.425	0.299	0.186
200	2.0	0.2	0.5	0.508	0.277	0.254
360				0.425	0.172	0.141
200	2.0	0.1	2.0	0.508	0.348	0.285
360				0.424	0.254	0.173
200	2.0	0.2	2.0	0.508	0.192	0.221
360				0.424	0.094	0.122

Note:  $(V_u/V_p)_N$  values in the table are calculated without considering the flange effect and  $k_0$  used is the AISC recommended value.

Table 5.5 - Test Results

Panel No.	Opening size, in. a o x b o	b t	a - b	Observed ult. load lb-force	Predicted Narayanan* ult. load lb-force	Predicted Narayanan* ult. load lb-force	Predicted M. Basler ult. load lb-force
Series A: Nominal Web Dimensions, a x b x t: 28.4 x 28.4 x 0.079 Nominal Flange Dimensions, b <sub>f</sub> x t <sub>f</sub> : 3.94 x 0.315							
NCP9	6.30 x 9.45	360	1 0.67	37250	28641	16620	6360
NCP9	9.45 x 9.45	360	1 1.00	35070	26303	15480	5580
NCP10	11.81 x 9.45	360	1 1.25	32283	25426	14670	5150
NCP10	14.17 x 9.45	360	1 1.50	30417	24055	13930	4750
RCP1	9.45 x 7.09	360	1 1.33	40600	32755	19370	6090
RCP2	9.45 x 14.17	360	1 0.67	25403	19603	82100	4780
Series B: Nominal Web Dimensions, a x b x t: 29.5 x 19.7 x 0.079 Nominal Flange Dimensions, b <sub>f</sub> x t <sub>f</sub> : 3.94 x 0.315							
NCP11	4.41 x 6.57	250	1.5 0.67	27786	22931	11167	7093
NCP11	6.57 x 6.57	250	1.5 1.00	22481	21964	10494	6520
NCP12	8.27 x 6.57	250	1.5 1.25	21155	21132	10010	6148
NCP12	9.84 x 6.57	250	1.5 1.50	22368	20268	95535	5811

NOTE: All the dimensions are in inches.

Narayanan\*:  $V_u/v_p$  values are calculated without considering the flange effect and  $k_o$  used is the AISC recommended value.

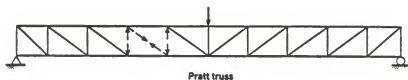


Figure 2.1 (a) Pratt Truss Analogy

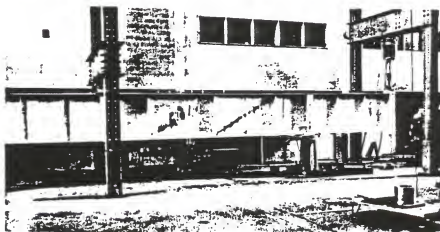
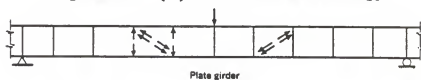


Figure 2.1 (b) Development of Tension Field in a Test Girder

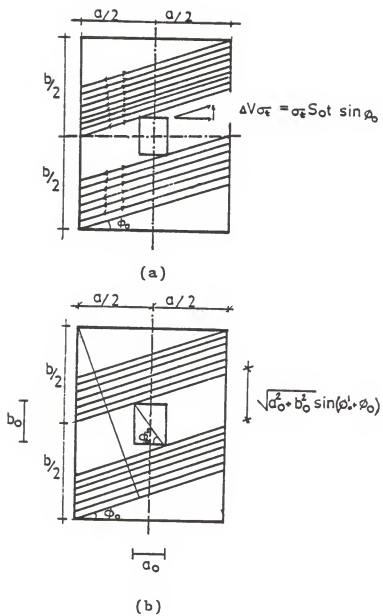


Figure 3.1 - Tension Field Action



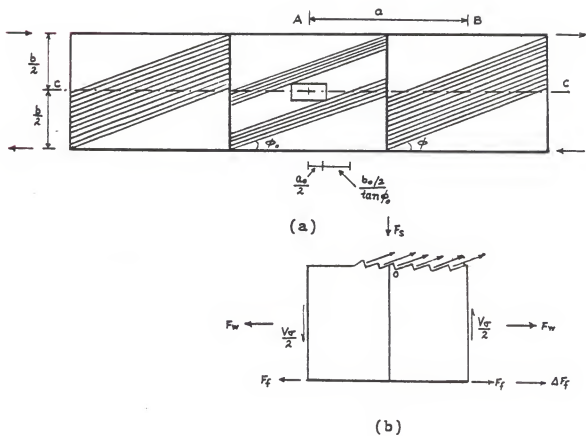


Figure 3.2 - Equilibrium Condition Applied to Free Body

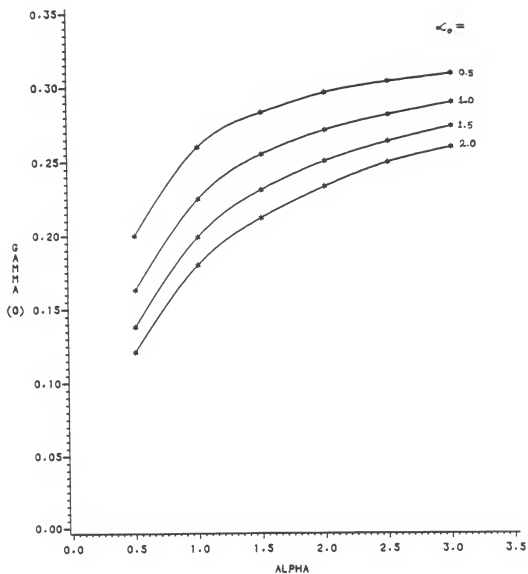


Figure 4.1 - Tension Field Limit Curves

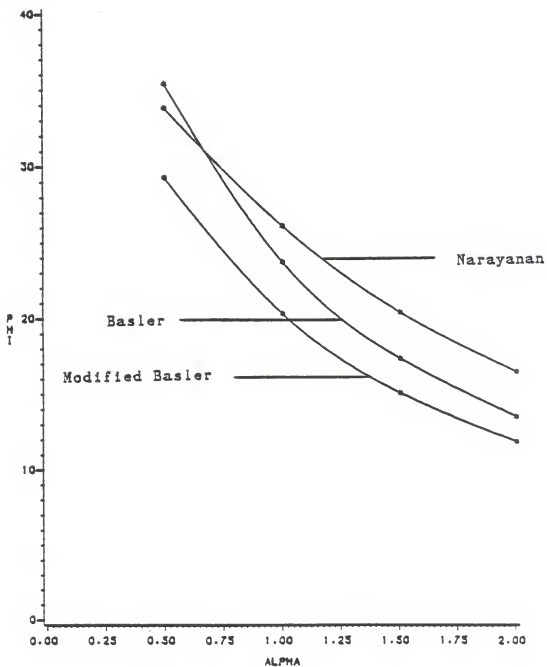


Figure 5.1 - Tension Field Inclination ( $\gamma_0 = 0.1$  ,  $\alpha_0 = 0.5$  )

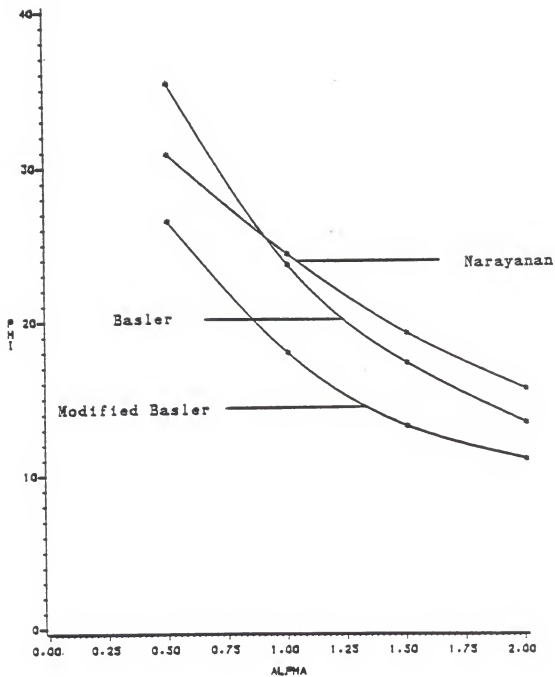


Figure 5.2 - Tension Field Inclination ( $\nu_o = 0.1$  ,  $\alpha_o = 0.5$  )

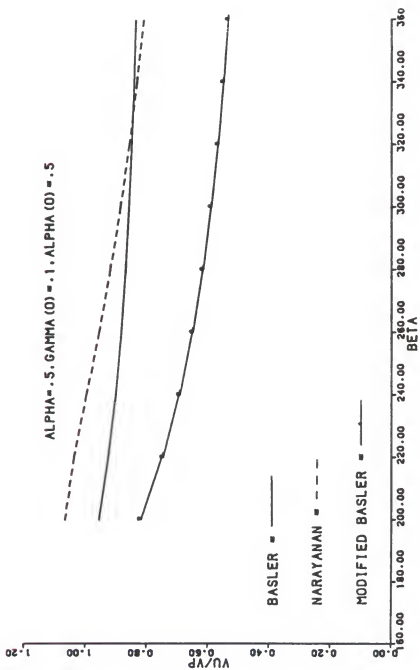


Figure 5.3 - Ultimate Shear Force ( $\alpha = 0.5$ ,  $\gamma_o = 0.1$ ,  $\alpha_o = 0.5$ )

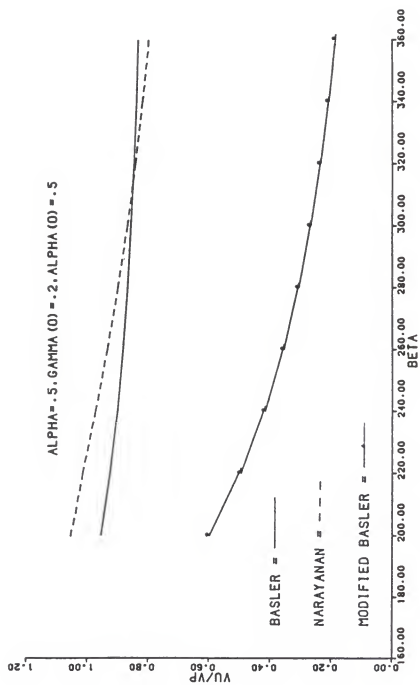


Figure 5.4 - Ultimate Shear Force ( $\alpha = 0.5$ ,  $\gamma_s = 0.2$ ,  $\alpha_0 = 0.5$ )

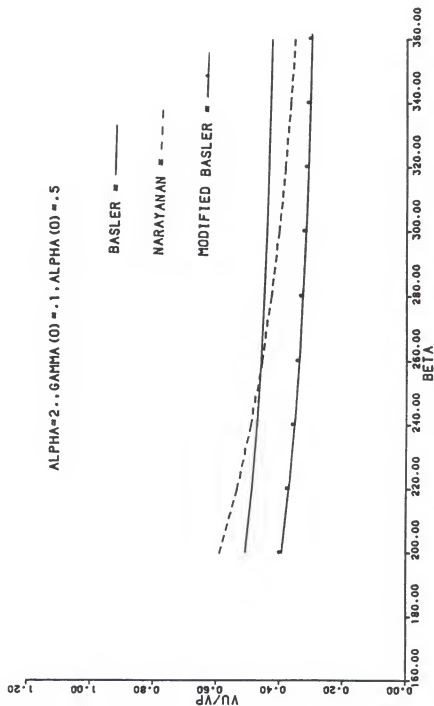


Figure 5.5 - Ultimate Shear Force ( $\alpha=2.0$ ,  $\gamma_0=0.1$ ,  $\alpha_0=0.5$ )

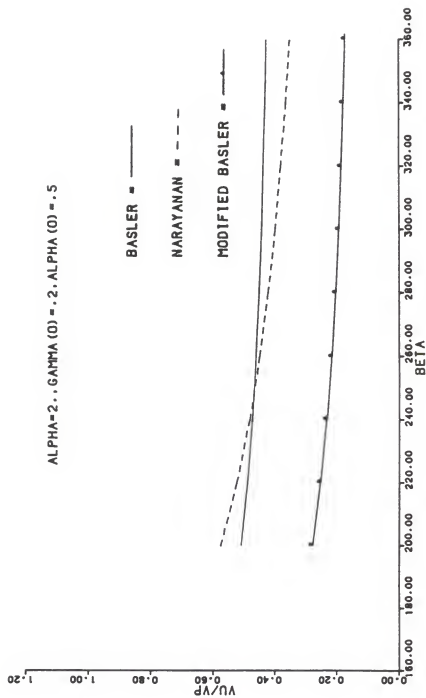


Figure 5.6 - Ultimate Shear Force ( $\alpha=2.0$ ,  $\gamma_o=0.2$ ,  $\alpha_o=0.5$ )



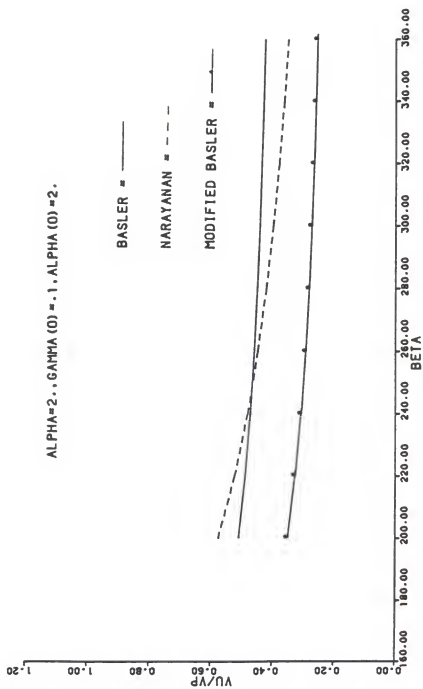


Figure 5.7 - Ultimate Shear Force ( $\infty=2.0, \gamma_0=0.1, \alpha_0=2.0$ )

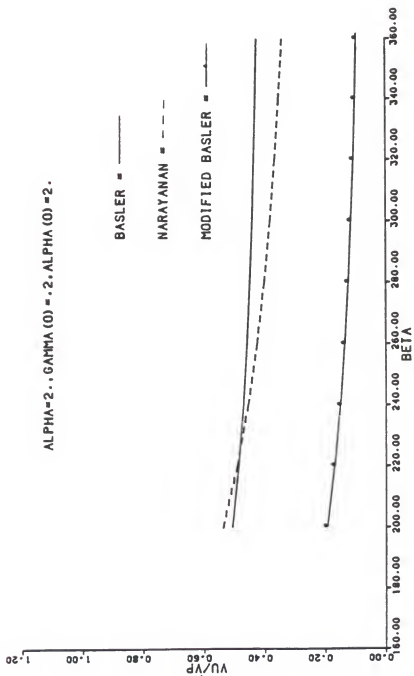


Figure 5.8 - Ultimate Shear Force ( $\alpha=2.0$ ,  $\gamma_0=0.2$ ,  $\alpha_0=2.0$ )

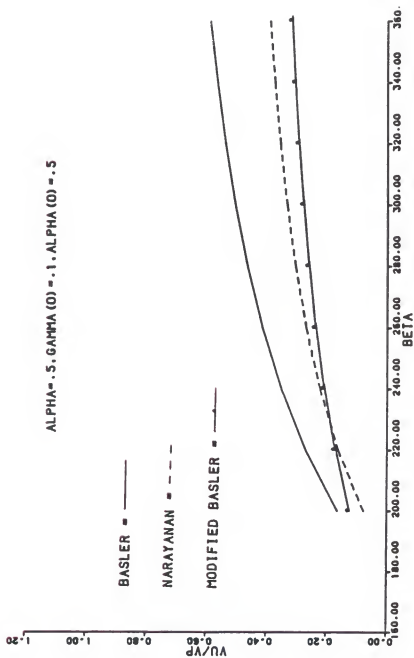


Figure 5.9 - Tension Field Force ( $\alpha=0.5$ ,  $\gamma_o=0.1$ ,  $\alpha_o=0.5$ )

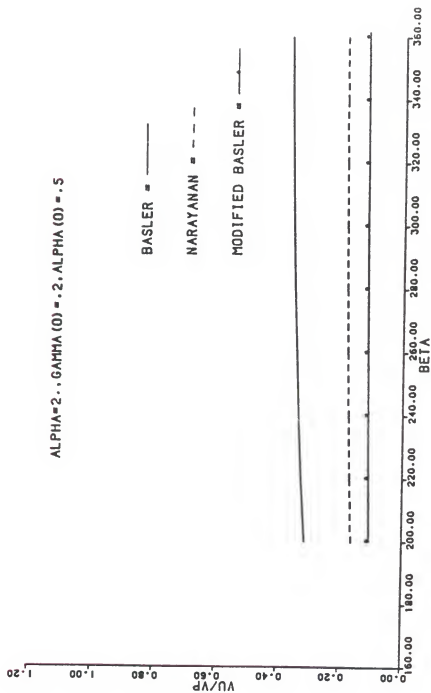


Figure 5.10 - Tension Field Force ( $\alpha = 2.0, \gamma_s = 0.2, \alpha_0 = 0.5$ )

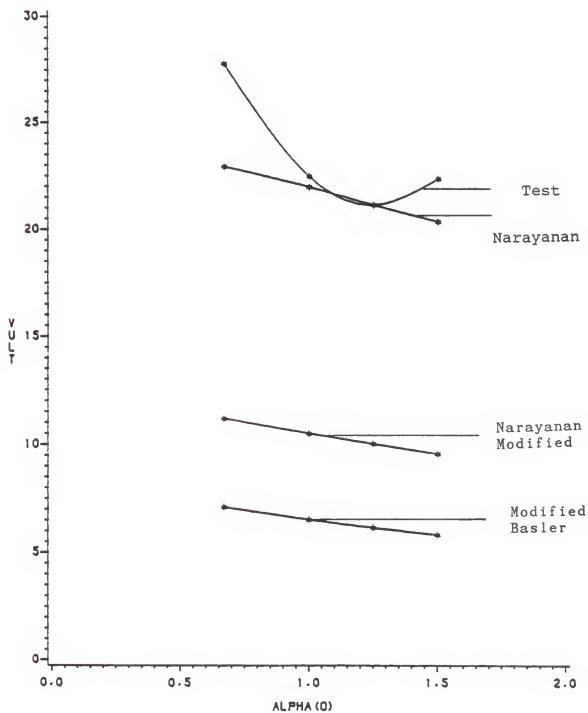


Figure 5.11 - Test Results

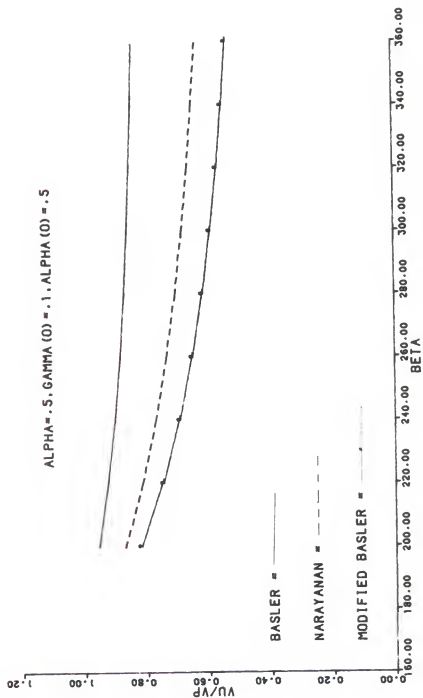


Figure 5.12 - Ultimate Shear Force (Narayanan Modified)

SHEAR STRENGTH OF PLATE GIRDERS  
WITH SMALL WEB OPENING

by

JYOTIRINDRA ROYCHOWDHURY

B.E., University of Gauhati, India, 1978

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AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the  
requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1987

## Abstract

The objective of this report is to obtain an expression for determination of shear strength of plate girders with small web opening. The method of analysis involved an approach similar to that followed by Konrad Basler, who stated the shear strength theory for plate girders without any web perforations. Results predicted by the modified Basler and Narayanan's theories are found to compare reasonably well. This theory is applicable to openings having area up to about 16% of the area of the web panel. Analytical results show that with the increase in aspect ratio the theory can be applied to larger openings.